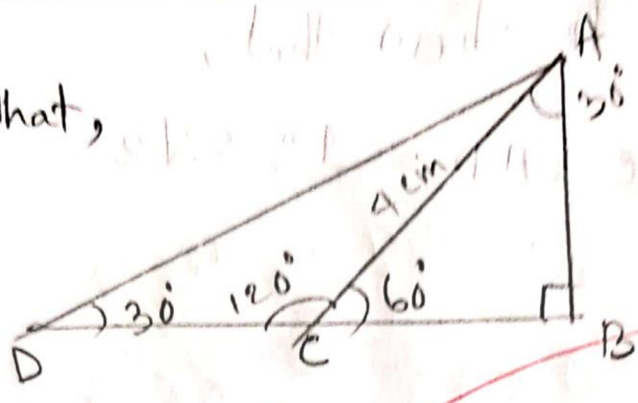
সাম্প্রতিক যুক্তি

Answer to Question NO: 11

(ক) Given that,



Then,

$\angle CAD = ?$

In triangle ABC, $\angle CAB = 180^\circ - \angle B - \angle C$

or, $\angle CAB = 180^\circ - 90^\circ - 60^\circ$

or, $\angle CAB = 30^\circ$

Now,

In triangle ~~ABD~~, ACD,

Exterior $\angle ACD = 180^\circ - 60^\circ$; $[\angle DCB = 180^\circ]$

$\cancel{= 130}$

$\therefore \angle ACD = 120^\circ$

Therefore, $\angle CAD = 180^\circ - \angle ADC - \angle ACD$

$= 180^\circ - 30^\circ - 120^\circ$

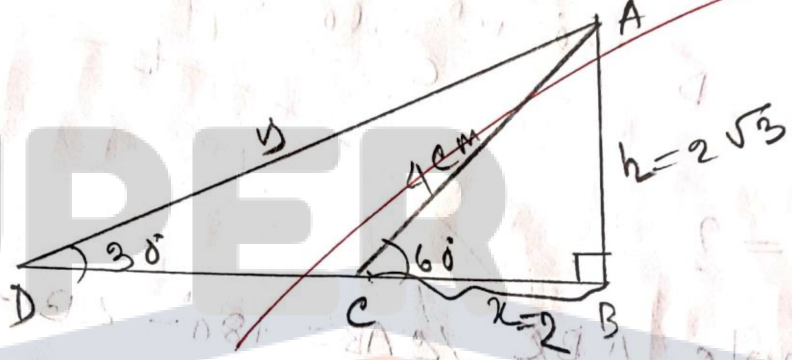
$= 180^\circ - 150^\circ$

$= 30^\circ$

$\therefore \angle CAD = 30^\circ$ $\boxed{\text{Ans.}}$

(25) We have to show that,

$$BC : AD = 1 : 2\sqrt{3}$$



L.H.S, $\frac{BC}{AD}$, Let, $BC = x$ and $AD = y$ and $AB = h$.

~~Now, BC =~~

$\cos 60^\circ = \frac{x}{4}$; $\left[\because \cos\theta = \frac{\text{base}}{\text{hypotenuse}}\right]$

or, $\frac{1}{2} = \frac{x}{4}$; $\left[\because \cos 60^\circ = \frac{1}{2}\right.$ and $AC = 4$ cm is given$\left.\right]$

or, $x = 2$

$\therefore BC = 2$ ---- (1)

Again, $AD = ?$

Now, from $\triangle ABC$, we get,

$$AB^2 = AC^2 - BC^2 \;; \quad [\because \text{pythagorean theorem}]$$

or, $h^2 = (4)^2 - (2)^2 \;; \quad [\because \text{from eq (1)} \; BC = 2]$

or, $(h)^2 = 16 - 4$

$\therefore h = \sqrt{12} = \sqrt{3 \times 4}$

$\therefore h = 2\sqrt{3}$ ---- (ii)

Therefore, $AD = \dfrac{AB}{\sin 30^\circ} \;; \quad \left[\because \sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenu}}\right]$

or, $\dfrac{AD}{AB} = \dfrac{1}{\sin 30^\circ}$

or, $\sin 30^\circ = \dfrac{AB}{AD} = \dfrac{2\sqrt{3}}{AD}$

or, $\dfrac{1}{2} = \dfrac{2\sqrt{3}}{AD} \;; \quad \left[\because \sin 30^\circ = \dfrac{1}{2}\right]$

or, $AD = 4\sqrt{3}$ ———— (iii)

From (i) and (iii)

$$\frac{BC}{AD} = \frac{2}{4\sqrt{3}}$$

or, $\frac{BC}{AD} = \frac{1}{2\sqrt{3}}$

or, $BC : AD = 1 : 2\sqrt{3}$.

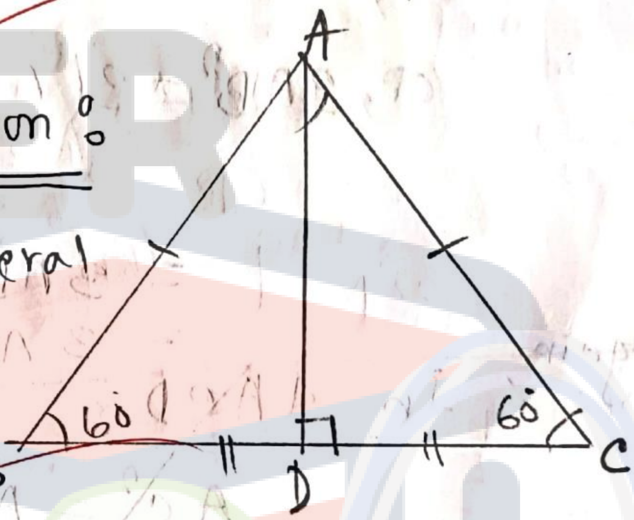
(Shown)

Answer to Question NO: 07

General Enunciation:

ABC is an equilateral triangle. AD is perpendicular to BC. Prove that $4AD^2 = 3AB^2$.

Particular Enunciation:



Let, ABC be an equilateral triangle of which AD is perpendicular to BC. And $AB = BC = CA$ as equilateral triangle.

we have to prove that $\boxed{4AD^2 = 3AB^2}$.

Proof: In triangles $\triangle ABD$ and $\triangle ACD$,

$AB = AC$; [given]

$AD = AD$; [common side]

$\angle ADB = \angle ADC = 90°$; [$\because AD \perp BC$]

$\therefore \triangle ABD \cong \triangle ACD$; [by SAS theorem]

$\therefore BD = CD = AD$ —— (i)

Now, In $\triangle ABD$,

$$AB^2 = AD^2 + BD^2 \text{ ; [by Pythagorean theorem]}$$

$$\text{or, } AB^2 = AD^2 + CD^2 \text{ ;}$$

$$\text{or, } 3AB^2 = 3(AD^2 + CD^2) \quad \text{—— (ii)}$$

$[\because BD = CD]$

$$= 3AD^2 + 3CD^2$$

$$= 3AD^2 + 3(AB^2 - BD^2) \quad \text{—— (ii)}$$

~~Again~~, In $\triangle ACD$,

$$AC^2 = AD^2 + CD^2 +$$

$$\text{or, } AB^2 = AD^2 + CD^2 \text{ ; } [\because AB = AC]$$

$$\text{or, } AD^2 = AB^2 - CD^2 \quad \text{—— (iii)}$$

$$\cancel{= A}$$

Now, L.H.S,

$$4AD^2$$

$$= 4(AB^2 - CD^2) \text{ ; } \because \cancel{AD^2 = AB^2 + C} \; [AD^2 = AB^2 - CD^2]$$

$$= 4AB^2 - 4CD^2$$

$= 4AB^2 - 4BD^2$; $[BD = CD]$

$= 4AB^2 - 4(AB^2 - AD^2)$

$= 4AB^2 - 4(AC^2 - AD^2)$; $[AB = AC$

$=$

$= \cancel{4 \cdot 4AB^2 - 4(}$

$= 4AB^2 - 4(AD^2 + CD^2 - AD^2)$

$= 4AB^2 - 4CD^2$

$= 4AB^2 -$

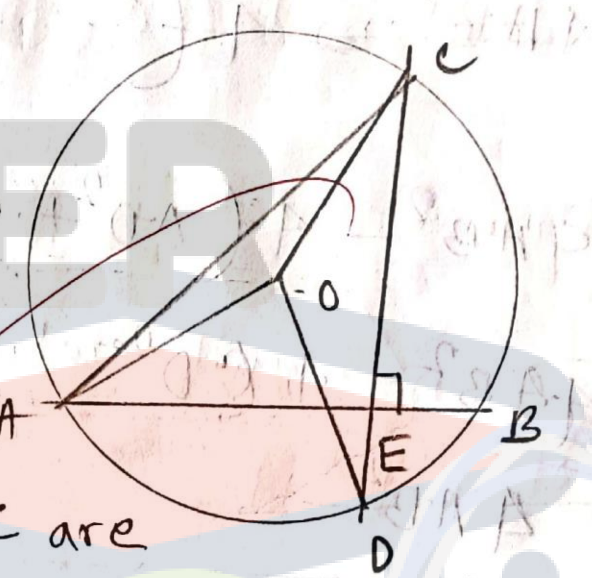
$3AB^2 = 3AD^2 + 3AB^2 - 3CD^2$

$=$

Answer to Question No: 08

General Enunciation:

A circle with O centre, has two chords; AB and ACD intersecting each other at point E and they intersect at 90° angle.



Prove that

$\angle AOD$ and $\angle BOC$ are ~~form~~ supplementary to each other.

Particular Enunciation:

Let ABC be the circle with O centre and two chords ~~AA~~ AB and CD intersecting each other at 90° at point E.

We have to prove that $\boxed{\angle AOD + \angle BOC = 180^\circ}$

<u>Construction:</u> Let us join O, A ; O, D ; C, A.

and O, C.

<u>Proof:</u> In arc AD,

~~peripheral~~

cyclic $\angle ACD = \frac{1}{2}$ central $\angle AOD$

$\therefore \angle ACD = \frac{1}{2} \angle AOD$ —— (i)

In arc BC,

Cyclic $\angle BAC = \frac{1}{2}$ central $\angle BOC$

$\therefore \angle BAC = \frac{1}{2} \angle BOC$ —— (ii)

Now, In $\triangle AEC$,

exterior $\angle AED$ = Opposite interior ($\angle DCA + \angle BAC$)

= Opposite interior ($\angle ACD + \angle BAC$)

—— (iii)

Now, from (i) and (ii),

$\angle ACD + \angle BAC = \frac{1}{2} \angle AOD + \frac{1}{2} \angle BOC$

or, exterior $\angle AED = \frac{1}{2}(\angle AOD + \angle BOC)$; [From equation (iii)] but $\angle AED = 90^\circ$; [$\because DC \perp AB$]

$\therefore 90^\circ = \frac{1}{2}(\angle AOD + \angle BOC)$

or, $180^\circ = (\angle AOD + \angle BOC)$

$\therefore \angle AOD + \angle BOC = 180^\circ$.

(proven)

Answer to Question – 09

Solution:

Given that, pth term of an arithme-
series = q. where a = 1st term and d =
-mor

$\therefore a + (p-1)d = q$ ——— (i) ; [as nth term $= a + (n-1)$

Similarly,
qth term;

$a + (q-1)d = p$ ——— (ii)

Therefore,

$$\text{mth term} = a + (m-1)d \quad \text{——— (iii)}$$

From (i) and (ii),

$$\cancel{a} + pd - \cancel{d} = q$$
$$\cancel{a} + qd - \cancel{d} = p$$
$$(-) \quad (-) \quad (+) \quad (-)$$

$$pd - qd = q - p$$

or, $d(p-q) = q - p$

or, $d = \frac{q-p}{p-q} = \frac{-1(p-q)}{(p-q)}$

$\therefore d = -1$

Now, putting d's value into equation (i),

~~a + pd − d~~ $\quad a + (p-1)d = q$

or, $a + pd - d = q$

or, $a + p(-1) - (-1) = q$

or, $a - p + 1 = q$

or, $a = q + p - 1$ ——— (iv)

Now, putting d and a's value into equation (iii),

$$
\begin{aligned}
m\text{th term} &= a + (m-1)d \\
&= (q + p - 1) + (m - 1)(-1) \\
&= p + q - 1 + m(-1) - (1)(-1) \\
&= p + q - 1 - m + 1 \\
&= p + q - m
\end{aligned}
$$

$m\text{th term} = \boxed{p + q - m}$ Ans.

Ans to Question : 06

(ক) L.H.S,

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$=\sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

2.5

$$=\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{\sin^2\theta+\cos^2\theta-\sin^2\theta}}$$

$$\cancel{=\sqrt{\frac{(1-\sin\theta)^2}{1-1-\cos^2\theta}}}$$

$$\cancel{= 1-2\sin\theta\cos\theta+\sin}$$

$$=\frac{\pm(1-\sin\theta)}{\cos\theta}$$

$$=\pm\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)$$

$$=\pm(\sec\theta-\tan\theta)$$

= R.H.S.

$\therefore \cos\theta = \cos 78°$; [$\because \cos 78° = 2$]

$\therefore \boxed{\theta = 78°}$ Ans'

Ans to Question - 10

Given that,

$A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6, 8\}$

$C = \{3, 4, 5, 6, 9\}$

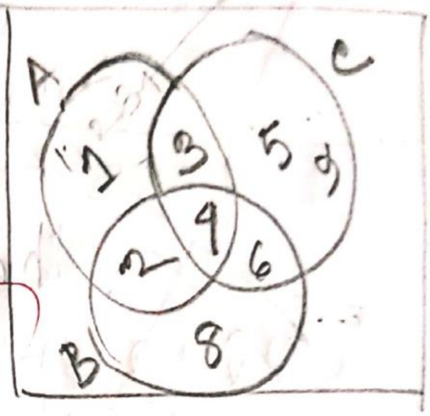


Fig = Venn Diagram.

L.H.S, $= n(A \cup B \cup C)$

$= n\{1, 2, 3, 4, 5, 6, 8, 9\}$

$= 8$

R.H.S,

$n(A) = 4$; [From diagram]

$n(B) = 4$

$n(C) = 5$

Fr venn diagram

$n(A \cap B) = 2$; [common element $= 2, 4$]

$n(B \cap C) = 2$; [common element $= 4, 6$]

$n(C \cap A) = 2$; [common element $= 3, 4$]

and $n(A \cap B \cap C) = 1$; [all common element = 9]

$\therefore$ R.H.S, (ii)

$n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 9 + 9 + 5 - 2 - 2 - 2 + 1$

$= \cancel{4 + 4 + 2 - 1 - 1 - 2 + 1}$

$= 13 - 6 + 1$

$= 14 - 6 = 8 = R.H.S.$

$\therefore$ L.H.S = R.H.S.

(proven.)

Answer to Question NO: 01

Given that,

$a+b+c=10$ ——(i)

$a^2+b^2+c^2=38$ ——(ii)

We know that,

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

or, $(10)^2=38+2(ab+bc+ca)$

or, $2(ab+bc+ca)=100-38$

$\therefore 2(ab+bc+ca)=62$ ——(iii)

Now, $(a-b)^2+(b-c)^2+(c-a)^2$

$=a^2-2ab+b^2+b^2-2bc+c^2+c^2-2ca+a^2$

$=2(a^2+b^2+c^2)-2(ab+bc+ca)$

$=2(38)-62$; [$\because$ from equation (iii)]

$=76-62$

$=14$. Ans

Answer to Question-02

L.H.S,

$$\frac{1}{\log_a(abc)} + \frac{1}{\log_b(abc)} + \frac{1}{\log_c(abc)}$$

$$= \frac{1}{\frac{1}{\log_{abc}(a)}} + \frac{1}{\frac{1}{\log_{abc}(b)}} + \frac{1}{\frac{1}{\log_{abc}(c)}}$$

$$= \log_{abc}(a) + \log_{abc}(b) + \log_{abc}(c)$$

$$= \log_{abc}(a \cdot b \cdot c) \; ; \left[\because \log_a(abc) = \log_a a + \log_a b + \log_a c\right]$$

~~= log~~

$$= 1 \; ; \left[\because \log_a a = 1\right]$$

$= R.H.S.$ (Shown)

Answer to Question - 09

Solution:

Given that,

pass in Bangla = 80 %.

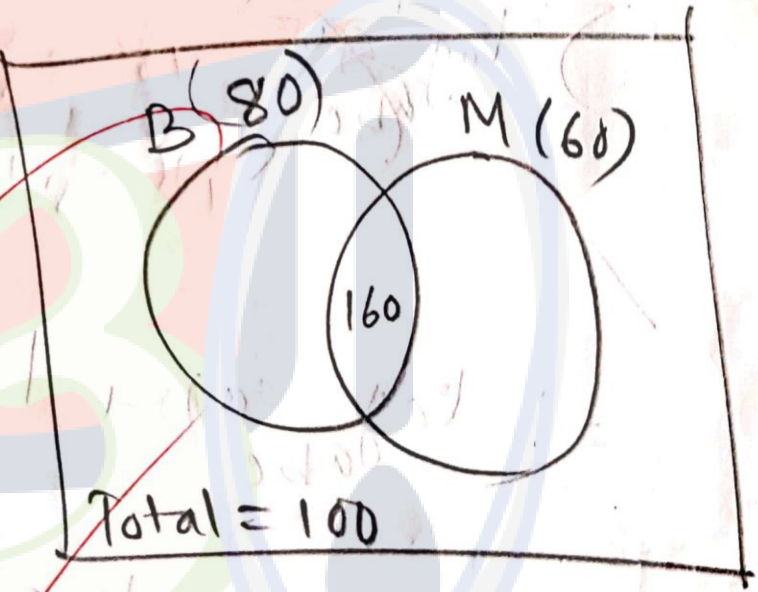
pass in Math = 60 %.

pass in Both = 160 people

Total students = ?

We know that,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$



Given that,

$n(A \cup B) = 100$

or, $100 = 80 + 60 - n(A \cap B)$

or, $100 - 190 = - n(A \cap B)$

or, $-90 = - n(A \cap B)$

$\therefore n(A \cap B) = 90$

Given that,

$$n(A \cap B) = n(B \cap M) = 160 \text{ people}$$

$$\therefore 40\% \text{ of total students} = 160$$

$$\text{or, } \frac{40}{100} \text{ of total students} = 160$$

$$\text{or, total students} = \frac{160 \times 100}{40}$$

$$= 4 \times 100$$

$$= 400$$

$$\therefore \boxed{\text{total students} = 400}$$

Ans

Answer to Question - 09

Solution:

Slope, $m = \frac{1}{2}$

Let (3, k) point be B.

So, According to the question the

Slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$

Where, $y_2 = k$, and $x_2 = 3$

$y_1 = 3$, $x_1 = -2$

$\therefore \frac{1}{2} = \frac{k-3}{3+2}$

or, $\frac{1}{2} = \frac{k-3}{5}$ or, $2(k-3) = 5$

or, $2k - 6 = 5$

$e-2$

or, $2k = 5+6 = 11$

$\therefore k = \dfrac{11}{2}$ (Ans)

Answer to Question No: 05

Soln: Given that,

$|x-1| < \frac{1}{10}$

or,